

Radiative Decays Involving $f_0(980)$ and $a_0(980)$

and

Mixing Between Low and High Mass Scalar Mesons

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Abstract

We analyze the experimental data for $\phi \rightarrow f_0(980)\gamma$, $\phi \rightarrow a_0(980)\gamma$, $f_0(980) \rightarrow \gamma\gamma$ and $a_0(980) \rightarrow \gamma\gamma$ decay widths in a framework where $f_0(980)$ and $a_0(980)$ are assumed to be mainly $qq\bar{q}\bar{q}$ low mass scalar mesons and mixed with $q\bar{q}$ high mass scalar mesons. Applied the vector meson dominance model (VDM), these decay amplitudes are expressed by coupling parameters B describing the $S(qq\bar{q}\bar{q}$ scalar meson)-V(vector meson)-V(vector meson) coupling and B' describing the $S'(q\bar{q}$ scalar meson)-V-V coupling. Adopting the magnitudes for B and B' as $\sim 2.8\text{GeV}^{-1}$ and $\sim 12\text{GeV}^{-1}$, respectively, the mixing angle between $a_0(980)$ and $a_0(1450)$ as $\sim 9^\circ$, and the mixing parameter λ_{01} causing the mixing between $I = 0$ $qq\bar{q}\bar{q}$ state and $q\bar{q}$ state as $\sim 0.24\text{GeV}^2$, we can interpret these experimental data, consistently.

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I. INTRODUCTION

From the recent experimental and theoretical analyses for the $f_0(600)$ and $\kappa(900)$, these scalar mesons are considered as the light scalar nonet together with the $f_0(980)$ and $a_0(980)$ [1]. From the status that though the masses of $f_0(980)$ and $a_0(980)$ are degenerate, the $f_0(980)$ state has the strangeness flavor rich character, many authors suggest this nonet as non $q\bar{q}$ structure, e. g. $K\bar{K}$ molecule [2], $qq\bar{q}\bar{q}$ state [3]. On this standpoint, many literatures assume that the higher mass scalar mesons, $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$, $f_0(1500)$ and $f_0(1710)$ would be traditional $q\bar{q}$ nonet and glueball state.

In order to confirm the structure of the light scalar mesons, that is whether the light scalar mesons are constituted of $q\bar{q}$ or $qq\bar{q}\bar{q}$, the radiative decays of the ϕ meson to the scalar mesons $f_0(980)$ and $a_0(980)$, $\phi \rightarrow f_0\gamma$ and $\phi \rightarrow a_0\gamma$, have long been analyzed by assuming the model, in which the decay $\phi \rightarrow f_0(a_0)\gamma$ proceeds through the charged K loop, $\phi \rightarrow K^+K^- \rightarrow f_0(a_0)\gamma$ [4]. Almost literature suggests that $f_0(980)$ and $a_0(980)$ mesons contain significant $qq\bar{q}\bar{q}$ content, specifically being $(u\bar{u} \pm b\bar{b})s\bar{s}$.

The two γ decays of the $f_0(980)$ and $a_0(980)$ mesons are also analyzed by using the various models, the linear sigma model [5], vector meson dominance model (VDM) [6] and the quark-hadron duality idea [7]. Literature [5] suggested that the $f_0(980)$ meson is mostly composed of $s\bar{s}$ component under the picture of the nonet scalar for light scalar mesons $f_0(600)$, $\kappa(900)$, $a_0(980)$ and $f_0(980)$. Literature [6] studied the $a_0(980) \rightarrow \gamma\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ decays and further $\phi \rightarrow f_0(980)\gamma$ and $\phi \rightarrow a_0(980)\gamma$ decays comprehensively assuming the VDM and the $qq\bar{q}\bar{q}$ structure for light scalar mesons. It could explain the experimental results for radiative decays except for the $\phi \rightarrow f_0(980)\gamma$ decay. Literature [7] considered the nonet scalar for light scalar mesons and assumed that these were composed of $q\bar{q}$ or $qq\bar{q}\bar{q}$ states. The author analyzed the two γ decays considering the mixing between $qq\bar{q}\bar{q}$ and $q\bar{q}$, latter of which is the dominant structure of the higher mass scalar mesons .

We analyzed the mixing between the light scalar nonet, $f_0(600)$, $\kappa(900)$, $a_0(980)$, $f_0(980)$ assumed as $qq\bar{q}\bar{q}$ states dominantly and high mass scalar nonet + glueball, $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$, $f_0(1500)$, $f_0(1710)$ assumed as $L = 1$ $q\bar{q}$ states dominantly + glueball state, in our previous work [8]. The estimated mixing is very strong because of the fact that the high mass scalar meson masses are very high compared with the masses supposed from the $L = 1$ $q\bar{q}$ 1^{++} and 2^{++} meson masses and relation $m^2(2^{++}) - m^2(1^{++}) =$

$2(m^2(1^{++}) - m^2(0^{++}))$ resulting from the $L \cdot S$ force. Literature [9] took the similar conclusion to ours in the mixing for $I = 1$ mesons and $I = 1/2$ mesons. That the mixing between $qq\bar{q}\bar{q}$ state and $q\bar{q}$ state is strong is recognized from the fact that the transition between $qq\bar{q}\bar{q}$ and $q\bar{q}$ states is caused by the OZI rule allowed diagram. In next work [10], we pursued this problem analyzing the decay processes in which light scalar mesons and high mass scalar meson decays to two pseudoscalar mesons, and get the result that the mixing angle between $I = 1$ $a_0(980)$ and $a_0(1450)$ is $\sim 10^\circ$.

In the present work, we will analyze the $\phi(1020) \rightarrow a_0(980)\gamma$, $\phi(1020) \rightarrow f_0(980)\gamma$, $a_0(980) \rightarrow \gamma\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ decays assuming that the light scalar mesons have the $qq\bar{q}\bar{q}$ component and $q\bar{q}$ component, and we will reveal the mixing ratio of these components. We analyze this problem using the vector dominance model, wherein we can treat these radiative decay processes comprehensively.

II. MIXING BETWEEN LOW AND HIGH MASS SCALAR MESONS

In this section, we briefly review the mixing among the low mass scalar, high mass scalar and glueball discussed in our previous work [8, 10]. The $qq\bar{q}\bar{q}$ scalar $SU(3)$ nonet S_a^b are represented by the quark triplet q_a and anti-quark triplet \bar{q}^a as [3], [9]

$$S_b^a \sim \epsilon^{acd} q_c q_d \epsilon_{bef} \bar{q}^e \bar{q}^f \quad (1)$$

and have the following flavor configuration:

$$\begin{aligned} \bar{s}\bar{d}us, \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds - \bar{s}\bar{u}us), \bar{s}\bar{u}ds &\iff a_0^+, a_0^0, a_0^- \\ \bar{s}\bar{d}ud, \bar{s}\bar{u}ud, \bar{u}\bar{d}us, \bar{u}\bar{d}ds &\iff \kappa^+, \kappa^0, \overline{\kappa^0}, \kappa^- \\ \frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us) &\iff f_{NS} \sim f_0(980) \\ \bar{u}\bar{d}ud &\iff f_{NN} \sim f_0(600) \end{aligned}$$

We use the notation f_{NS} for $\frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us)$ and f_{NN} for $\bar{u}\bar{d}ud$ in this paper, but we used f_N and f_S for $\frac{1}{\sqrt{2}}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us)$ and $\bar{u}\bar{d}ud$, respectively in our previous literature [8, 10]. The high mass scalar mesons $S_b'^a$ are the ordinary $SU(3)$ nonet

$$S_b'^a \sim \bar{q}^a q_b.$$

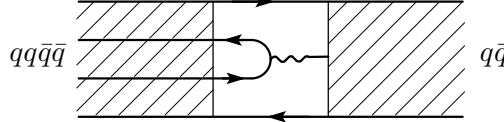


Fig.1 OZI rule allowed graph for $qq\bar{q}\bar{q}$ and $q\bar{q}$ states transition

A. Inter-mixing between $I = 1, 1/2$ low and high mass scalar mesons

The mixing between $qq\bar{q}\bar{q}$ and $q\bar{q}$ states, for which we call "inter-mixing", may be large, because the transition between $qq\bar{q}\bar{q}$ and $q\bar{q}$ states is caused by the OZI rule allowed diagram shown in fig. 1. This transition is represented as

$$L_{\text{int}} = -\lambda_{01}\epsilon^{abc}\epsilon_{def}N_a^dN_b^e\delta_c^f = \lambda_{01}[a_0^+a_0'^- + a_0^-a_0'^+ + a_0^0a_0'^0 + \kappa^+K_0^{*-} + \kappa^-K_0^{*+} + \kappa^0K_0^{*0} + \bar{\kappa}^+\bar{K}_0^{*-} - \sqrt{2}f_Nf'_N - f_Sf'_N - \sqrt{2}f_Nf'_S]. \quad (2)$$

The inter-mixing parameter λ_{01} represents the strength of the inter-mixing and can be considered as rather large.

We first consider the $I = 1$ $a_0(980)$ and $a_0(1450)$ mixing. Representing the before mixing $qq\bar{q}\bar{q}$ state by $\overline{a_0(980)}$ and $q\bar{q}$ by $\overline{a_0(1450)}$ and mixing angle as θ_a , physical after mixing state $a_0(980)$ and $a_0(1450)$ are written as follows;

$$\begin{aligned} a_0(980) &= \cos\theta_a\overline{a_0(980)} - \sin\theta_a\overline{a_0(1450)} \\ a_0(1450) &= \sin\theta_a\overline{a_0(980)} + \cos\theta_a\overline{a_0(1450)} \end{aligned} \quad (3)$$

and the mixing matrix is represented as

$$\begin{pmatrix} m_{\overline{a_0(980)}}^2 & \lambda_{01}^a \\ \lambda_{01}^a & m_{\overline{a_0(1450)}}^2 \end{pmatrix}, \quad (4)$$

where $m_{\overline{a_0(980)}}$ and $m_{\overline{a_0(1450)}}$ are the before mixing masses for $a_0(980)$ and $a_0(1450)$ states. For the values of $m_{\overline{a_0(980)}}$ and $m_{\overline{a_0(1450)}}$, in the first our work [8], we adopted the values

$$m_{\overline{a_0(980)}} = 1271 \pm 31 \text{ MeV}, \quad m_{\overline{a_0(1450)}} = 1236 \pm 20 \text{ MeV},$$

estimated from the relation $m^2(2^{++}) - m^2(1^{++}) = 2(m^2(1^{++}) - m^2(0^{++}))$ resulting from the $L \cdot S$ force. Diagonalising the mass matrix in Eq. (4) and taking the eigenvalues of masses

$$m_{a_0(980)} = 984.8 \pm 1.4 \text{ MeV}, \quad m_{a_0(1450)} = 1474 \pm 19 \text{ MeV}, \quad (5)$$

we can get the result

$$\lambda_{01}^a = 0.600 \pm 0.028 \text{GeV}^2, \quad \theta_a = 47.1 \pm 3.5^\circ. \quad (6)$$

Similarly, we estimated the strength λ_{01}^K and mixing angle θ_K for $I = 1/2$ $\kappa(900)$ and $K_0^*(1430)$ mixing case. Using the mass values before mixing and after mixing,

$$\begin{aligned} m_{\overline{\kappa(900)}} &= 1047 \pm 62 \text{MeV}, & m_{\overline{K_0^*(1430)}} &= 1307 \pm 11 \text{MeV}, \\ m_{\kappa(900)} &= 900 \pm 70 \text{MeV}, & m_{K_0^*(1430)} &= 1412 \pm 6 \text{MeV}, \end{aligned} \quad (7)$$

we get the results

$$\lambda_{01}^K = 0.507 \pm 84 \text{GeV}^2, \quad \theta_K = 29.5 \pm 15.5^\circ. \quad (8)$$

In our next work [10], we estimated the mixing angle θ_a and θ_K analyzing the $a_0(980)$, $a_0(1450)$ and $K_0^*(1430)$ decay processes to two pseudoscalar meson, and get the results,

$$\theta_a = \theta_K = (9 \pm 4)^\circ. \quad (9)$$

The value of λ_{01} and mass values before mixing of $a_0(980)$, $a_0(1450)$ and $\kappa(900)$, $K_0^*(1430)$ for this mixing angle are estimated as follows;

$$\begin{aligned} \lambda_{01}^a &= \lambda_{01}^K = 0.19^{+0.07}_{-0.09} \text{GeV}^2, \\ m_{\overline{a_0(980)}} &= 1.00^{+0.02}_{-0.01} \text{GeV}, & m_{\overline{a_0(1450)}} &= 1.46 \pm 0.01 \text{GeV}, \\ m_{\overline{K_0(900)}} &= 0.92 \pm 0.01 \text{GeV}, & m_{\overline{K_0^*(1430)}} &= 1.40 \pm 0.01 \text{GeV}. \end{aligned} \quad (10)$$

B. Inter-mixing between $I = 0$ low and high mass scalar mesons

Among the $I = 0, L = 1$ $q\bar{q}$ scalar mesons, there are the intra-mixing weaker than the inter-mixing, caused from the transition between themselves represented by the OZI rule suppression graph shown in Fig. 2, and furthermore the mixing between the $q\bar{q}$ scalar meson and the glueball caused from the transition represented by the graph shown in Fig. 3. Thus, the mass matrix for these $I = 0, L = 1$ $q\bar{q}$ scalar mesons and glueball is represented as

$$\begin{pmatrix} m_{N'}^2 + 2\lambda_1 & \sqrt{2}\lambda_1 & \sqrt{2}\lambda_G \\ \sqrt{2}\lambda_1 & 2m_{S'}^2 + \lambda_1 & \lambda_G \\ \sqrt{2}\lambda_G & \lambda_G & \lambda_{GG} \end{pmatrix}, \quad (11)$$

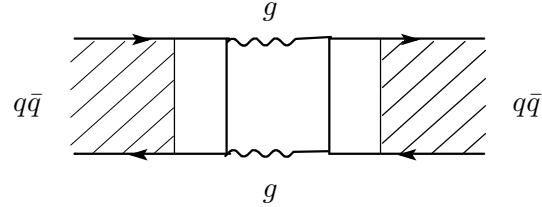


Fig. 2. OZI rule suppression graph for $q\bar{q} - q\bar{q}$ transition.

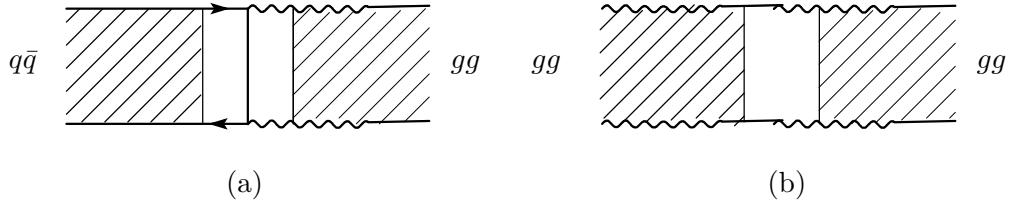


Fig. 3. Transition graph between (a) $q\bar{q}$ and gg , and (b) gg and gg .

where $m_{N'}^2 = m_{a'_0}^2$, $m_{S'}^2 = 2m_{K'_0}^2 - m_{a'_0}^2$, and λ_1 is the transition strength among the $I = 0$, $q\bar{q}$ mesons, λ_G is the transition strength between $q\bar{q}$ and glueball gg and λ_{GG} is the pure glueball mass square.

For the light $I = 0$ $qq\bar{q}\bar{q}$ scalar mesons, there are the intra-mixing caused from the transition between themselves represented by the OZI rule suppression graph shown in Fig. 4, and the mass matrix for these $I = 0$ $qq\bar{q}\bar{q}$ scalar meson is represented as

$$\begin{pmatrix} m_{NN}^2 + 2\lambda_0 & \sqrt{2}\lambda_0 \\ \sqrt{2}\lambda_0 & 2m_{NS}^2 + \lambda_0 \end{pmatrix}, \quad (12)$$

where $m_{NN}^2 = 2m_{K'_0}^2 - m_{a'_0}^2$, $m_{NS}^2 = m_{a'_0}^2$, and λ_0 represents the transition strength between $I = 0$ $qq\bar{q}\bar{q}$ mesons.

The inter- and intra-mixing among $I = 0$ low mass and high mass scalar mesons and

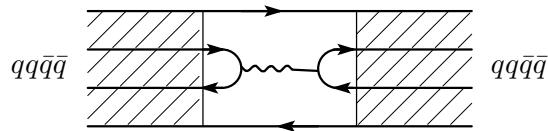


Fig. 4. OZI suppression graph for $qq\bar{q}\bar{q} - qq\bar{q}\bar{q}$ transition.

glueball is expressed by the overall mixing mass matrix as

$$\begin{pmatrix} m_{NN}^2 + \lambda_0 & \sqrt{2}\lambda_0 & \sqrt{2}\lambda_{01} & 0 & 0 \\ \sqrt{2}\lambda_0 & m_{NS}^2 + 2\lambda_0 & \lambda_{01} & \sqrt{2}\lambda_{01} & 0 \\ \sqrt{2}\lambda_{01} & \lambda_{01} & m_{N'}^2 + 2\lambda_1 & \sqrt{2}\lambda_1 & \sqrt{2}\lambda_G \\ 0 & \sqrt{2}\lambda_{01} & \sqrt{2}\lambda_1 & m_{S'}^2 + \lambda_1 & \lambda_G \\ 0 & 0 & \sqrt{2}\lambda_G & \lambda_G & \lambda_{GG} \end{pmatrix}. \quad (13)$$

We use the values of $\lambda_{01} = 0.19\text{GeV}^2$ tentatively, corresponding to the mixing angle $\theta_a = \theta_K = 9^\circ$ estimated in analyses of the decay processes as shown in Eq. (10), and then diagonalize this 5×5 mass matrix. In this case, we input the values for m_{NN} etc. as follows;

$$m_{NN} = 0.826\text{GeV}, \quad m_{NS} = 1.00\text{GeV}, \quad m_{N'} = 1.46\text{GeV}, \quad m_{S'} = 1.34\text{GeV}, \quad (14)$$

predicted from the relations, $m_{NN}^2 = 2m_{K_0}^2 - m_{a_0}^2$, $m_{NS}^2 = m_{a_0}^2$ and $m_{S'}^2 = 2m_{K'_0}^2 - m_{a'_0}^2$, $m_{N'}^2 = m_{a'_0}^2$ and the estimated values Eq. (10). Varying the parameters λ_0 , λ_1 , λ_G and λ_{GG} , we get the best fit eigenvalues for the mass of $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$,

$$\begin{aligned} m_{f_0(600)} &= 0.77(0.80 \pm 0.40)\text{GeV}, \quad m_{f_0(980)} = 0.93(0.980 \pm 0.010)\text{GeV}, \\ m_{f_0(1370)} &= 1.37(1.350 \pm 0.150)\text{GeV}, \quad m_{f_0(1500)} = 1.51(1.507 \pm 0.015)\text{GeV}, \\ m_{f_0(1710)} &= 1.71(1.714 \pm 0.005)\text{GeV}, \end{aligned} \quad (15)$$

where the values in parenthesis are quoted from [11]. Best-fit values are obtained for the values of λ_0 etc.,

$$\lambda_0 = -0.03\text{GeV}^2, \quad \lambda_1 = 0.04\text{GeV}^2, \quad \lambda_G = 0.1\text{GeV}^2, \quad \lambda_{GG} = 1.7^2\text{GeV}^2, \quad (16)$$

and at this time, mixing parameters are calculated as

$$\begin{pmatrix} f_0(600) \\ f_0(980) \\ f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = [R_{f_0(M)I}] \begin{pmatrix} f_{NN} \\ f_{NS} \\ f_{N'} \\ f_{S'} \\ f_G \end{pmatrix}, \quad (17)$$

$$[R_{f_0(M)I}] = \begin{pmatrix} 0.949 & 0.250 & -0.185 & -0.047 & 0.013 \\ 0.270 & -0.925 & 0.067 & 0.257 & -0.017 \\ 0.054 & -0.233 & 0.210 & -0.946 & 0.064 \\ 0.150 & 0.161 & 0.932 & 0.160 & -0.239 \\ 0.025 & 0.035 & 0.220 & 0.107 & 0.969 \end{pmatrix}.$$

The mixing parameters $[R_{f_0(M)I}]$ for $f_0(600)$ and $f_0(980)$ are similar to those obtained in our previous works [8], but the ones for $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ are rather different from them. This comes about from the smaller values of λ_{01} in the present analysis. Because only the mixing parameters $[R_{f_0(M)I}]$ of $f_0(980)$ and mixing angle θ_a between $a_0(980)$ and $f_0(1450)$ are necessary in this work, the deviation of the mixing parameters $[R_{f_0(M)I}]$ for $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ from previous result does not get into trouble.

III. RADIATIVE DECAYS INVOLVING $F_0(980)$ AND $a_0(980)$

A. Radiative decays involving $f_0(980)$ and $a_0(980)$ in VDM

In this work, we analyze the radiative decays involving $f_0(980)$ and $a_0(980)$; $\phi(1020) \rightarrow f_0(980)\gamma$, $\phi(1020) \rightarrow a_0(980)\gamma$ and $f_0(980) \rightarrow \gamma\gamma$, $f_0(980) \rightarrow \gamma\gamma$. We assume the vector meson dominance model (VDM) in this analysis, for the radiative processes involving pseudoscalar mesons, $P \rightarrow \gamma\gamma$, $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ have been interpreted well in VDM [12]. In VDM, $V \rightarrow S\gamma$ and $S \rightarrow \gamma\gamma$ processes are described by the diagram shown in Fig.5.

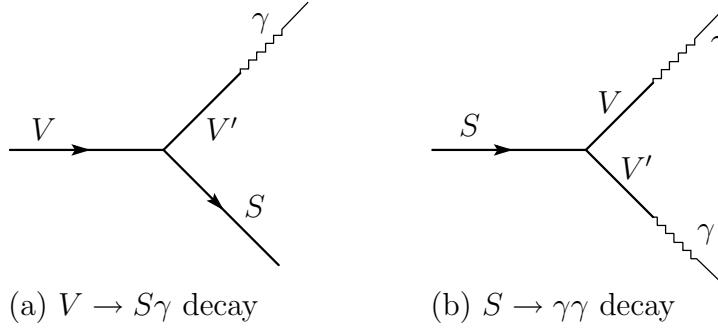


Fig. 5. Diagram for $V \rightarrow S\gamma$ and $S \rightarrow \gamma\gamma$ decays in VDM

We use the following interactions for SVV , $S'VV$ and GVV coupling with coupling constants B , B' and B'' , respectively,

$$L_I = B\varepsilon^{abc}\varepsilon_{def}S_a^dV^{\mu\nu e}V_{\mu\nu}^f + B'S_a^b\{V^{\mu\nu c}, V_{\mu\nu}^a\} + B''G\{V^{\mu\nu b}, V_{\mu\nu}^a\}, \quad (18)$$

where $V^{\mu\nu}$ is the vector field strength $\partial^\mu V^\nu - \partial^\nu V^\mu$. These interactions are represented graphically by the diagrams shown in fig. 6. Although interactions as $\text{Tr}(SV_{\mu\nu})\text{Tr}(V^{\mu\nu})$ and $\text{Tr}(S'V_{\mu\nu})\text{Tr}(V^{\mu\nu})$ other than those represented by Eq. (18) may exist [9], these interactions violate the *OZI* rule and are considered to be smaller than the interactions in Eq. (18).

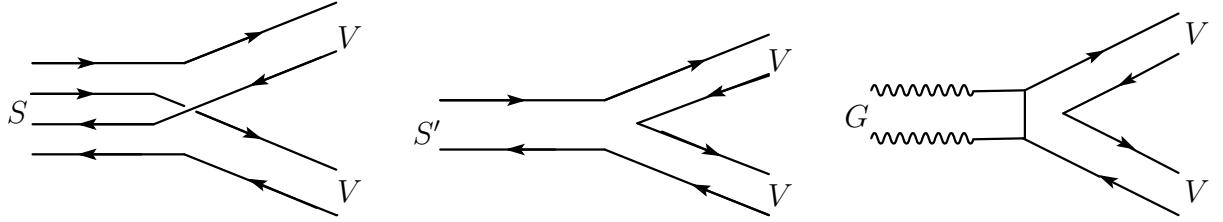


Fig.6. SVV , $S'VV$ and GVV coupling

We define the coupling constants $g_{SVV'}$ in the following expression,

$$\begin{aligned} L_I = & g_{a_0 K^* \bar{K}^*} \frac{1}{\sqrt{2}} \overline{K}_{\mu\nu}^* \boldsymbol{\tau} \cdot \mathbf{a}_0 K^{*\mu\nu} + g_{a'_0 K^* \bar{K}^*} \frac{1}{\sqrt{2}} \overline{K}_{\mu\nu}^* \boldsymbol{\tau} \cdot \mathbf{a}'_0 K^{*\mu\nu} + g_{a_0 \rho \omega} \mathbf{a}_0 \cdot \boldsymbol{\rho}_{\mu\nu} \omega^{\mu\nu} \\ & + g_{a'_0 \rho \omega} \mathbf{a}'_0 \cdot \boldsymbol{\rho}_{\mu\nu} \omega^{\mu\nu} + g_{a_0 \rho \phi} \mathbf{a}_0 \cdot \boldsymbol{\rho}_{\mu\nu} \phi^{\mu\nu} + g_{a'_0 \rho \phi} \mathbf{a}'_0 \cdot \boldsymbol{\rho}_{\mu\nu} \phi^{\mu\nu} \\ & + g_{K_0^* K^* \rho} \left(\frac{1}{\sqrt{2}} \overline{K}_{\mu\nu}^* \boldsymbol{\tau} \cdot \boldsymbol{\rho}^{\mu\nu} K_0^* + H.C. \right) + g_{\kappa K^* \kappa} \left(\frac{1}{\sqrt{2}} \overline{K}_{\mu\nu}^* \boldsymbol{\tau} \cdot \boldsymbol{\rho}^{\mu\nu} \kappa + H.C. \right) \\ & + g_{\kappa K^* \omega} (\overline{\kappa} K_{\mu\nu}^* \omega^{\mu\nu} + H.C.) + g_{K_0^* K^* \omega} (\overline{K}_0^* K_{\mu\nu}^* \omega^{\mu\nu} + H.C.) \\ & + g_{\kappa K^* \phi} (\overline{\kappa} K_{\mu\nu}^* \phi^{\mu\nu} + H.C.) + g_{K_0^* K^* \phi} (\overline{K}_0^* K_{\mu\nu}^* \phi^{\mu\nu} + H.C.), \\ & + g_{f_0(M) \rho \rho} f_0(M) \boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + g_{f_0(M) K^* \bar{K}^*} f_0(M) K_{\mu\nu}^* \overline{K}^{*\mu\nu} + g_{f_0(M) \omega \omega} f_0(M) \omega_{\mu\nu} \omega^{\mu\nu} \\ & + g_{f_0(M) \omega \phi} f_0(M) \omega_{\mu\nu} \phi^{\mu\nu} + g_{f_0(M) \phi \phi} f_0(M) \phi_{\mu\nu} \phi^{\mu\nu}, \end{aligned} \quad (19)$$

where fields a_0 represents the low mass $I = 1$ scalar mesons and a'_0 the high mass $I = 1$ scalar mesons. Then the coupling constants for $I_3 = 0$ S, V mesons are, by using Eq. (18), expressed as

$$\begin{aligned}
g_{a_0(980)\rho\phi} &= -2B \cos \theta_a, \\
g_{a_0(980)\rho\omega} &= -2\sqrt{2}B' \sin \theta_a, \\
g_{f_0(980)\phi\omega} &= 2BR_{f_0(980)NS}, \\
g_{f_0(980)\phi\phi} &= 2B'R_{f_0(980)S'} + 2B''R_{f_0(980)G}, \\
g_{f_0(980)\rho\rho} &= -BR_{f_0(980)NN} + \sqrt{2}B'R_{f_0(980)N'} + 2B''R_{f_0(980)G}, \\
g_{f_0(980)\omega\omega} &= BR_{f_0(980)NN} + \sqrt{2}B'R_{f_0(980)N'} + 2B''R_{f_0(980)G}.
\end{aligned} \tag{20}$$

$V - \gamma$ coupling is defined as

$$<0|j_\mu^{\text{em}}(0)|V(p, \varepsilon)> = \frac{em_V^2}{\gamma_V} \varepsilon_\mu(p), \tag{21}$$

where m_V and $\varepsilon_\mu(p)$ are the mass and polarization vector of the vector meson, respectively. Then $\Gamma(V \rightarrow ll) = \frac{4\pi}{3} \frac{\alpha^2}{\gamma_V^2} m_V$, we can get the value $\frac{1}{\gamma_\rho} = 0.201 \pm 0.002$ from the experimental data $\Gamma(\rho \rightarrow ee) = 7.02 \pm 0.11 \text{ keV}$ [11].

The decay amplitude and decay width for $V \rightarrow S\gamma$ are expressed as

$$\begin{aligned}
M(V \rightarrow S\gamma) &= \sum_{V'} \frac{2eg_{SVV'}}{\gamma_{V'}} (p \cdot k \varepsilon_V \cdot \varepsilon_\gamma - p \cdot \varepsilon_\gamma k \cdot \varepsilon_V), \\
\Gamma(V \rightarrow S\gamma) &= \frac{4\alpha}{3} \left(\sum_{V'} \frac{g_{SVV'}}{\gamma_{V'}} \right)^2 |\mathbf{k}_\gamma|^3,
\end{aligned} \tag{22}$$

where ε_V and ε_γ are the polarization vector of the vector meson and the photon, respectively and \mathbf{k}_γ is the photon momentum in the V rest frame. Only the ρ meson contributes to the intermediate V' vector meson for the $\phi \rightarrow a_0(980)\gamma$ decay, and ω and ϕ mesons contribute to $\phi \rightarrow f_0(980)\gamma$ decay. Then the decay widths for the $\phi \rightarrow a_0(980)\gamma$ and $\phi \rightarrow f_0(980)\gamma$ decay are written as

$$\begin{aligned}
\Gamma(\phi \rightarrow a_0(980)\gamma) &= \frac{4\alpha}{3\gamma_\rho^2} (g_{a_0\rho\phi})^2 |\mathbf{k}_\gamma|^3, \\
\Gamma(\phi \rightarrow f_0(980)\gamma) &= \frac{4\alpha}{3\gamma_\rho^2} \left(\frac{g_{f_0\phi\omega}}{3} - \frac{\sqrt{2}g_{f_0\phi\phi}}{3} \right)^2 |\mathbf{k}_\gamma|^3,
\end{aligned} \tag{23}$$

where we assumed the $SU(3)$ symmetry for the $V - \gamma$ coupling,

$$\frac{m_\rho^2}{\gamma_\rho} : \frac{m_\omega^2}{\gamma_\omega} : \frac{m_\phi^2}{\gamma_\phi} \cong \frac{1}{\gamma_\rho} : \frac{1}{\gamma_\omega} : \frac{1}{\gamma_\phi} = \frac{1}{\sqrt{2}} : \frac{1}{3\sqrt{2}} : -\frac{1}{3}.$$

For the decay $S \rightarrow \gamma\gamma$, decay amplitude and width are expressed as

$$\begin{aligned} M(S \rightarrow \gamma\gamma) &= \sum_{VV'} \frac{2e^2 g_{SVV'}}{\gamma_V \gamma_{V'}} (k_1 \cdot k_2 \varepsilon_1 \cdot \varepsilon_2 - k_1 \cdot \varepsilon_2 k_2 \cdot \varepsilon_1), \\ \Gamma(S \rightarrow \gamma\gamma) &= \pi \alpha^2 m_S^3 \left(\sum_{VV'} \frac{g_{SVV'}}{\gamma_V \gamma_{V'}} \right)^2, \end{aligned} \quad (24)$$

where k_1 , k_2 and ε_1 , ε_2 are the photon momentums and polarization vectors of photon. The decay widths for $a_0(980) \rightarrow \gamma\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ are expressed as

$$\begin{aligned} \Gamma(a_0(980) \rightarrow \gamma\gamma) &= \frac{\pi \alpha^2}{\gamma_\rho^2} m_{a_0}^3 \left(-\frac{\sqrt{2}}{3} g_{a_0\rho\phi} + \frac{1}{3} g_{a_0\rho\omega} \right)^2, \\ \Gamma(f_0(980) \rightarrow \gamma\gamma) &= \frac{\pi \alpha^2}{\gamma_\rho^2} m_{f_0}^3 \left(-\frac{\sqrt{2}}{9} g_{f_0\phi\omega} + g_{f_0\rho\rho} + \frac{1}{9} g_{f_0\omega\omega} + \frac{2}{9} g_{f_0\phi\phi} \right)^2. \end{aligned} \quad (25)$$

B. Numerical Analysis

We quote the experimental data for these decay widths from PDG [11],

$$\begin{aligned} \Gamma(\phi \rightarrow a_0(980)\gamma) &= 0.323 \pm 0.029 \text{ keV}, \\ \Gamma(\phi \rightarrow f_0(980)\gamma) &= 1.87 \pm 0.11 \text{ keV}, \\ \Gamma(a_0(980) \rightarrow \gamma\gamma) &= 0.30 \pm 0.10 \text{ keV}, \\ \Gamma(f_0(980) \rightarrow \gamma\gamma) &= 0.39^{+0.10}_{-0.13} \text{ keV}. \end{aligned} \quad (26)$$

The coupling strengths for these radiative decays are described by the coupling constants B , B' abd B'' from the equations (20), (23) and (25), and the values for these coupling strengths are determined by the experimental data Eq. (26) as

$$\begin{aligned} |B \cos \theta_a| &= 2.316 \pm 0.127 \text{ GeV}^{-1}, \text{ from } \phi \rightarrow a_0(980)\gamma \text{ decay} \\ |BR_{f_0NS} - \sqrt{2}B'R_{f_0S'} - \sqrt{2}B''R_{f_0G}| &= 13.8 \pm 0.54 \text{ GeV}^{-1}, \text{ from } \phi \rightarrow f_0(980)\gamma \text{ decay} \\ |B \cos \theta_a - B' \sin \theta_a| &= 1.137 \pm 0.214 \text{ GeV}^{-1}, \text{ from } a_0(980) \rightarrow \gamma\gamma \text{ decay} \\ |B(R_{f_0NS} + 2\sqrt{2}R_{f_0NN}) - B'(5R_{f_0N'} + \sqrt{2}R_{f_0S'}) - 6\sqrt{2}B''R_{f_0G}| &= 3.920^{+0.591}_{-0.789} \text{ GeV}^{-1}, \\ &\text{from } f_0(980) \rightarrow \gamma\gamma \text{ decay}. \end{aligned} \quad (27)$$

Because only the 4 quark component of $a_0(980)$ can contribute to the $\phi \rightarrow a_0(980)\gamma$ decay, one can not explain the $\phi \rightarrow a_0(980)\gamma$ decay width without the 4-quark model for the low mass scalar mesons. The experimental fact that the coupling strength of $\phi \rightarrow f_0(980)\gamma$

decay is larger as several factor than that of $\phi \rightarrow a_0(980)\gamma$ can be explained by the mixing of the 2 quark state in the $f_0(980)$ meson, $\sqrt{2}B'R_{f_0S'}$ factor. If we take the relative sign of the B' to the B as same, $\sqrt{2}B'R_{f_0S'}$ factor contributes additively by the opposite sign of the R_{f_0NS} to the $R_{f_0S'}$ (see Eq. (17)). The coupling strength for $a_0(980) \rightarrow \gamma\gamma$ is a half of that for $\phi \rightarrow f_0(980)\gamma$. This can be explained easily by the fact that B and B' have the same sign and θ_a is positive, then $B'\sin\theta_a$ term contribute destructively. The term $BR_{f_0NS} - \sqrt{2}B'R_{f_0S'} - \sqrt{2}B''R_{f_0G}$ in the coupling of $f_0(980) \rightarrow \gamma\gamma$ decay is the same as that in $\phi \rightarrow f_0(980)\gamma$ decay. The term $2\sqrt{2}BR_{f_0NN}$ in the coupling of $f_0(980) \rightarrow \gamma\gamma$ decay contributes destructively because of the opposite sign of the mixing parameter R_{f_0NN} to that of the R_{f_0NS} (see the eq. (17)).

We got the best fit values of coupling constant B , B' and B'' taking the χ^2 fit analysis for the experimental data and these coupling constants contained in Eq. (27). Because the coupling strengths contain the mixing parameters R_{a_0NS} etc., we vary the values of R_{f_0NS} etc. in the χ^2 analysis. Variations of the mixing parameters R_{a_0NS} etc. are caused by the shift of the values of λ_{01} , λ_0 , λ_1 , λ_{GG} , λ_G , then we perform the χ^2 fit varying these parameters. The best fit values of the coupling strengths for radiative decays are obtained and shown in Table I. These values are given on the following values of coupling constants B etc. and mixing mass parameters λ_{01} etc.;

$$\begin{aligned} B &= 2.8\text{GeV}^{-1}, \quad B' = 12.0\text{GeV}^{-1}, \quad B'' = 7.2\text{GeV}^{-1}, \\ \lambda_{01} &= 0.24\text{GeV}^2, \quad \lambda_0 = -0.03\text{GeV}^2, \quad \lambda_1 = 0.09\text{GeV}^2, \\ \lambda_G &= 0.17\text{GeV}^2, \quad \lambda_{GG} = 1.66^2\text{GeV}^2. \end{aligned} \quad (28)$$

At this time, obtained mass eigenvalues and mixing parameters for $f_0(980)$ etc. are as follows;

$$m_{f_0(600)} = 0.748\text{GeV}, \quad m_{f_0(980)} = 0.912\text{GeV}, \quad m_{f_0(1370)} = 1.381\text{GeV},$$

$$m_{f_0(1500)} = 1.522\text{GeV}, \quad m_{f_0(1710)} = 1.711\text{GeV},$$

$$\begin{pmatrix} f_0(600) \\ f_0(980) \\ f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.929 & 0.292 & -0.218 & -0.058 & 0.028 \\ 0.315 & -0.901 & 0.053 & 0.292 & -0.032 \\ 0.088 & -0.251 & 0.295 & -0.912 & 0.099 \\ 0.160 & 0.178 & 0.808 & 0.172 & -0.510 \\ 0.067 & 0.092 & 0.458 & 0.222 & 0.853 \end{pmatrix} \begin{pmatrix} f_{NN} \\ f_{NS} \\ f_{N'} \\ f_{S'} \\ f_G \end{pmatrix}. \quad (29)$$

TABLE I: Experimental data [11] and best-fit values of coupling strength for radiative decay involving $a_0(980)$ and $f_0(980)$. These best-fit values are obtained on the values of coupling and mass parameters; $B = 2.8\text{GeV}^{-1}$, $B' = 12.0\text{GeV}^{-1}$, $B'' = 7.2\text{GeV}^{-1}$, $\lambda_{01} = 0.24\text{GeV}^2$, $\lambda_0 = -0.03\text{GeV}^2$, $\lambda_1 = 0.09\text{GeV}^2$, $\lambda_G = 0.17\text{GeV}^2$, $\lambda_{GG} = 1.66^2\text{GeV}^2$.

Decay	Coupling strength	Experimental data	Best-fit value
$\phi \rightarrow a_0(980)\gamma$	$ B \cos \theta_a $	$2.316 \pm 0.127 \text{ GeV}^{-1}$	2.76 GeV^{-1}
$\phi \rightarrow f_0(980)\gamma$	$ BR_{f_0 NS} - \sqrt{2}B'R_{f_0 S'} - \sqrt{2}B''R_{f_0 G} $	$13.8 \pm 0.54 \text{ GeV}^{-1}$	7.15 GeV^{-1}
$a_0(980) \rightarrow \gamma\gamma$	$ B \cos \theta_a - B' \sin \theta_a $	$1.137 \pm 0.214 \text{ GeV}^{-1}$	0.84 GeV^{-1}
$f_0(980) \rightarrow \gamma\gamma$	$ B(R_{f_0 NS} + 2\sqrt{2}R_{f_0 NN}) - B'(5R_{f_0 N'} + \sqrt{2}R_{f_0 S'}) - 6\sqrt{2}B''R_{f_0 G} $	$3.920^{+0.591}_{-0.789} \text{ GeV}^{-1}$	6.20 GeV^{-1}

We can explain the radiative decays, $\phi \rightarrow a_0(980)\gamma$, $\phi \rightarrow f_0(980)\gamma$, $a_0(980) \rightarrow \gamma\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ comprehensively in the VDM model, assuming that the low mass scalar $a_0(980)$ and $f_0(980)$ mesons are composed of $qq\bar{q}\bar{q}$ state dominantly and there exists the mixing between the low mass scalar and high mass $q\bar{q}$ scalar mesons. The literature [6] studied these decays in the VDM systematically, but it could not explain the experimental large decay width of $\phi \rightarrow f_0(980)\gamma$. To explain this large decay width of $\phi \rightarrow f_0(980)\gamma$, the work [13] applied the a_0 - f_0 mixing. But that the a_0 - f_0 mixing effect for the $\phi \rightarrow f_0(980)\gamma$ decay is small was shown by the authors N. N. Achasov and A. V. Kiselev [14]. We explain this large decay width of $\phi \rightarrow f_0(980)\gamma$ by the $qq\bar{q}\bar{q}$ scalar and $q\bar{q}$ scalar meson mixing.

Although we can make the coupling strength for $\phi \rightarrow f_0(980)\gamma$ so large, the best-fit value of this is a half of the experimental data. On the other hand, the best fit value of the coupling strength for $f_0(980) \rightarrow \gamma\gamma$ is 1.6 times of the experimental data. In our model, the coupling strengths $BR_{f_0 NS} - \sqrt{2}B'R_{f_0 S'} - \sqrt{2}B''R_{f_0 G}$ are common in the couplings of $\phi \rightarrow f_0(980)\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ decay and the residual term $2\sqrt{2}BR_{f_0 NN} - 5B'R_{f_0 N'} - 5\sqrt{2}B''R_{f_0 G}$ in the coupling for $f_0(980) \rightarrow \gamma\gamma$ are not so large, then the best fit values of these radiative decays are not so different. In order to explain the rather large difference of experimental data between $\phi \rightarrow f_0(980)\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ decay, any other process than that deduced from the VDM may be considered.

In our present reanalysis for radiative decays of scalar mesons, we can get the best

fit values for mixing parameters R_{f_0NS} etc. and mass eigenvalues for $f_0(980)$ meson etc. Obtained mass values 0.912GeV for $f_0(980)$ is rather small compared with the experimental value 0.980 ± 0.010 GeV. This mass value of $f_0(980)$ is affected with the input values of the masses for $m_{f_0(600)}$ and $m_{\kappa(900)}$ and these input masses are very uncertain at present, then we may get the mass eigenvalue for $f_0(980)$ nearer to the experimental values 0.980 ± 0.010 GeV by sifting the input mass values for $f_0(600)$ and $\kappa(900)$.

IV. CONCLUSION

We analyzed the radiative decays $\phi \rightarrow f_0(980)\gamma$, $\phi \rightarrow a_0(980)\gamma$, $f_0(980) \rightarrow \gamma\gamma$ and $a_0(980) \rightarrow \gamma\gamma$ using the VDM in the framework where $f_0(980)$ and $a_0(980)$ are mainly $qq\bar{q}\bar{q}$ scalar mesons and are mixed with $q\bar{q}$ high mass scalar mesons. Mixing between $a_0(980)$ and $a_0(1450)$ has been studied in our previous paper [10], where decay processes of $a_0(980)$ and $a_0(1450)$ decaying to two pseudoscalar mesons are analyzed, and the mixing angle θ_a has estimated to be about 9° . The mass mixing parameter λ_{12} causing the mixing between $qq\bar{q}\bar{q}$ state and $q\bar{q}$ state corresponds to values to be 0.19 GeV 2 . In present work, we estimated the mixing parameters $R_{f_0(980)NS}$ etc. performing the χ^2 analysis varying the λ_{12} near the values ~ 0.19 GeV 2 .

We assumed the form of the SVV coupling $B\varepsilon^{abc}\varepsilon_{def}S_a^dV^{\mu\nu e}_bV^{\mu\nu f}_c + B'S_a'^b\{V^{\mu\nu c}_b, V_{\mu\nu c}^a\} + B''G\{V^{\mu\nu b}_a, V_{\mu\nu b}^a\}$ similar to that for SPP coupling, which was used in our previous work [10]. By assuming the VDM, the coupling strengths for $\phi \rightarrow a_0(980)\gamma$, $\phi \rightarrow f_0(980)\gamma$, $a_0(980) \rightarrow \gamma\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ decays are expressed as $|B \cos \theta_a|$, $|BR_{f_0NS} - \sqrt{2}B'R_{f_0S'} - \sqrt{2}B''R_{f_0G}|$, $|B \cos \theta_a - B' \sin \theta_a|$ and $|B(R_{f_0NS} + 2\sqrt{2}R_{f_0NN}) - B'(5R_{f_0N'} + \sqrt{2}R_{f_0S'}) - 6\sqrt{2}B''R_{f_0G}|$, respectively. Adopting the magnitudes for B , B' and B'' as 2.8 GeV $^{-1}$, 12 GeV $^{-1}$ and 7.2 GeV $^{-1}$, respectively, we can get the values of the coupling strength for $\phi \rightarrow a_0(980)\gamma$, $\phi \rightarrow f_0(980)\gamma$, $a_0(980) \rightarrow \gamma\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ decays as 2.76 GeV $^{-1}$, 7.15 GeV $^{-1}$, 0.84 GeV $^{-1}$ and 6.20 GeV $^{-1}$, respectively. The experimental values for these coupling strengths are 2.316 ± 0.127 GeV $^{-1}$, 13.8 ± 0.54 GeV $^{-1}$, 1.137 ± 0.214 GeV $^{-1}$ and $3.920^{+0.591}_{-0.789}$ GeV $^{-1}$, then one can say that our model using the VDM and mixing between

$qq\bar{q}\bar{q}$ and $q\bar{q}$ can explain the radiative decays including $a_0(980)$ and $f_0(980)$ consistently.

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